Pre-Calculus

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Pre-Calculus Curriculum Overview

Grade 11/12 Pre-Calculus is taught in 10 units throughout the school year. The Pre-Calculus curriculum is a rigorous, open-ended and sequential process of connecting the trigonometric and calculus material to previously learned algebra topics. As part of the spiraling curriculum, aspects of Algebra, Geometry and Algebra II are taught throughout the year. A guided inquiry program gives students the opportunity to explore topics and concepts through mathematical investigations. Participating in this rigorous program helps students:

1. **To foster a lifelong enjoyment of learning mathematics.**
2. **To observe mathematics in the real world around them.**
3. **To meet the mathematics standards for New Jersey Public Schools.**

Course Sequence:

Unit 1 Functions & Graphs *22 days*

Unit 2 Polynomial & Rational Functions *20 days*

Unit 3 Exponential & Logarithmic Functions *16 days*

Unit 4 Trigonometric Functions *26 days*

Unit 5 Analytic Trigonometry *18 days*

Unit 6 Polar Coordinates & Vectors *16 days*

Unit 7 Systems of Equations & Matrices *22 days*

Unit 8 Conic Sections *16 days*

Unit 9 Sequences & Series *12 days*

Unit 10 Introduction to Calculus *14 days*

Pre-requisite:
- Algebra II

Unit 1 - Overview

Content Area: Pre-Calculus

Unit Title: Functions & Graphs

Grade Level: 11 – 12

Core Ideas: Students will use functions to model real-world scenarios. Students will use prior knowledge from Algebra II to expand their understanding of functions, including piecewise functions, graphical transformations of functions, combinations of functions, and the inverse of functions. Students will use properties of functions to determine continuity and end behavior of functions.

Unit 1 - Standards

Standards (Content and Technology)**:**

CPI#:	Statement:
Performance Expectations (NJSLS)	
A-CED.A.	Create equations that describe numbers or relationships.
A-CED.A.1.	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
A-CED.A.2.	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
F-IF.A.	Understand the concept of a function and use function notation.
F-IF.A.1.	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
F-IF.A.2.	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F-IF.B.	Interpret functions that arise in applications in terms of the context.
F-IF.B.4.	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
F-IF.B.5.	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(x) gives the number of person hours it takes to assemble n engines in a factory, then the positive*

	integers would be an appropriate domain for the function.
F-IF.B.6.	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
F-IF.C.	Analyze functions using different representations.
F-IF.C.7.	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
F-IF.C.7.a.	Graph linear and quadratic functions and show intercepts, maxima, and minima.
F-IF.C.7.b.	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
F-IF.C.7.c.	Graph polynomial functions, identifying zeros when suitable factorizations are available.
F-IF.C.7.d.	(+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
F-IF.C.8.	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F-IF.C.8.a.	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
F-IF.C.9.	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
F-BF.A.	Build a function that models a relationship between two quantities.
F-BF.A.1.	Write a function that describes a relationship between two quantities.
F-BF.A.1.a.	Determine an explicit expression, a recursive process, or steps for calculation from a context.
F-BF.A.1.b.	Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
F-BF.A.1.c.	(+) Compose functions. *For example, if $\blacklozenge(\blacklozenge)$ is the temperature in the atmosphere as a function of height, and h($\blacklozenge$) is the height of a weather balloon as a function of time, then $\blacklozenge$,h($\blacklozenge$)- is the temperature at the location of the weather balloon as a function of time.*
F-BF.B.	Build new functions from existing functions.

F-BF.B.3.	Identify the effect on the graph of replacing �(�) by �(�) + �, � · �(�), �(��), and �(� + �) for specific values of � (both positive and negative); find the value of � given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
F-BF.B.4.	Find inverse functions.
F-BF.B.4.a.	Solve an equation of the form �(�) = � for a simple function � that has the inverse and write an expression for the inverse. *For example,* $\mathbf{\diamond}(\mathbf{\diamond}) = 2\mathbf{\diamond}^!$ *or* $\mathbf{\diamond}(\mathbf{\diamond}) = \frac{"\#\$}{"\%\$}$ *for* $\mathbf{\diamond} \neq 1.$
F-BF.B.4.b.	(+) Verify by composition that one function is the inverse of another.
F-BF.B.4.c.	(+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
F-BF.B.4.d.	(+) Produce an invertible function from a non-invertible function by restricting the domain.
Career Readiness, Life Literacies, and Key Skills	
9.4.12.CI.1.	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CI.3.	Investigate new challenges and opportunities for personal growth, advancement, and transition.
9.4.12.CT.2.	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.TL.4.	Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem.
Computer Science and Design Thinking	
8.1.12.DA.5.	Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
8.1.12.DA.6.	Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.1.12.AP.1.	Design algorithms to solve computational problems using a combination of original and existing algorithms.
8.2.12.ED.6.	Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor).
Intercultural Statements (Amistad, Holocaust, LGBT, etc…)	

LGBTQ and Disabilities NJSA 18A:35-4.35	Explore mathematicians in the LGBTQ community, including but not limited to Juliette Bruce, NSF Postdoctoral Fellow at University of California, Berkeley, and Stephen Hawking, former Director of Research at the University of Cambridge.
Amistad Law NJSA 18A 52:16A-88	Explore African American mathematicians and scientists, including but not limited to Martha Euphemia Lofton Haynes, the first African American woman to earn a Ph.D mathematics, and Elbert Frank Cox, the first African American man to earn a Ph.D in mathematics in the world.
Amistad Law NJSA 18A 52:16A-88	Discuss and analyze the movie *Hidden Figures,* the story of female African American mathematicians and engineers who worked for NASA.
Holocaust Law NJSA 18A 18A:35-28	Explore Jewish mathematicians using the article *"Jewish Mathematicians Who Changed the Course of History"* from jewishjournal.com.
AAPI Law NJSA 18A:25-4.44	Explore Asian-American and Pacific Islander mathematicians and scienctists, including but not limited to Dr. Peter Tsai, inventor of the N95 respirator and Diana Ma, data scientist for the Lakers.
Interdisciplinary Connection	
6.1.12.EconE M.6.a.	Determine how supply and demand influenced price and output during the Industrial Revolution.
6.1.12.EconN E.6.a.	Analyze the impact of money, investment, credit, savings, debt, and financial institutions on the development of the nation and the lives of individuals.
6.1.12.EconG E.16.a.	Use quantitative data and other sources to assess the impact of international trade, global business organizations, and overseas competition on the United States economy and workforce.

Social Emotional Learning

Self-Awareness: The abilities to understand one's own emotions, thoughts, and values and how they influence behavior across contexts.
- Having a growth mindset
- Developing interests and a sense of purpose

Social Awareness: The abilities to understand the perspectives of and empathize with others, include those from diverse backgrounds, cultures, & contexts.
- Recognizing strengths in others
- Recognizing situational demands and opportunities

Self-Management: The abilities to manage one's emotions, thoughts, and behaviors effectively in different situations and to achieve goals and aspirations.
- Identifying and using stress-management strategies
- Exhibiting self-discipline and self-motivation
- Setting personal and collective goals
- Using planning and organizational skills

Relationship Skills: The abilities to establish and maintain healthy and supportive relationships and to effectively navigate settings with diverse individuals and groups.
- Practicing teamwork and collaborative problem-solving
- Showing leadership in groups
- Seeking or offering support and help when needed

Companion Standards

NJSLSA.R7.	Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.

RH-11-12.7.	Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, qualitatively, as well as in words) in order to address a question or solve a problem.

RST.11-12.3.	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.

Unit Essential Question(s):
 • How can you represent and describe functions? •
   How can functions describe real-world situations,
      model predictions, and solve problems?
 • How can you describe a function that is
      represented by more than one
      equation?
 • How does the graph of $g(x)$ compare to
      the parent graph, $f(x)$?
 • How are a function and its inverse related?

Unit Enduring Understandings:
 • Functions are a mathematical way to describe
      relationships between two quantities that
      vary.
 • Functions can be represented in a variety of
      ways (graphically, numerically,
      algebraically, or verbally).
 • A function and its inverse are closely related
      graphically and algebraically.
 • The properties of functions, function
      operations, and function compositions
      are used to model and analyze
      real-world
   applications and quantitative relationships.

Evidence of Learning

Formative Assessments:
  • Quizzes
  • Homework
  • On-the-spot check for understanding activities
  • Entry / Exit ticket
Summative/Benchmark Assessment(s):
  • Quizzes
  • Tests
  • Projects
Alternative Assessments:
  • Notebook & note checks

Resources/Materials:	Key Vocabulary:
• All Things Algebra – Precalculus Curriculum • Jean Adams – Precalculus Curriculum • *Larson Precalculus with Limits* textbook	• Relation • Function • Domain • Range • Function notation • Implicit form of a function • Explicit form of a function • Vertical line test • Intercepts • Local / relative extrema • Global / absolute extrema • Boundedness (lower bound and upper bound) • Symmetry of a function • Even and odd functions • Concavity • Inflection point • Continuity

	• Removable discontinuity
	    o Point discontinuity
	    o Hole
	• Nonremovable discontinuity
	    o Jump discontinuity
	    o Infinite discontinuity
	• Asymptote
	    o Horizontal
	    o Vertical
	    o Slant (oblique)
	• End behavior
	• Average rate of change
	• Piecewise function
	• Absolute value function
	• Ceiling function
	• Floor function
	• Constant function
	• Identity function
	• Rational function
	• Quadratic function
	• Cubic function
	• Square root function
	• Absolute value function
	• Cube root function
	• Exponential function
	• Logarithmic function
	• Greatest integer function
	• Signum function
	• Sine function, cosine function, tangent function, cotangent function, cosecant function, secant function (all introduced, but not studied until Unit 4)
	• Transformations of functions (both rigid and non-rigid)
	• Dilation
	• Reflection
	• Rotation
	• Translation
	• Function combinations
	• Function compositions
	• One-to-one function
	• Horizontal line test
	• Inverse function
	• Rational function
	• Cost function
	• Revenue function
	• Profit function
	• Break-even point

Suggested Pacing Guide			
Lesson Name/Topi c	**Student Learning Objective(s)**	**Suggested Tasks/Activities:**	**Day(s) to Complete**
1.1 Function Basics	• Determine whether a relation represents a function. • Find the domain and range of a function. • Use function notation. • Evaluate a function using function notation. • Use and convert between explicit and implicit forms of a function. • Use functions in real-world applications.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
1.2 Graphs of Functions	• Use the vertical line test to determine if a curve is a graph of a function or not. • Find x- and y-intercepts; determine on what intervals a function is positive or negative. • Determine intervals on which a function is increasing, decreasing, or constant. • Identify local / relative and global / absolute extrema. • Determine relative maximum and minimum values. • Determine if a function is bounded above, bounded below, or bounded both above and below. • Identify even and odd functions. • Determine intervals of concavity. • Find points of inflection.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2

1.3 Properties of Functions	• Determine if a function is continuous. • Identify different types of discontinuities (removable vs. nonremovable). • Find and use horizontal, vertical, and slant asymptotes to analyze the behavior of a graph. • Find the end behavior of functions.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
	• Find the average rate of change of a function on an interval. • Compare functions to each other given multiple representations (graphically, numerically, algebraically, and verbally). • Interpret graphs. • Perform function operations given multiple representations.		
1.4 Piecewise Functions	• Evaluate piecewise functions. • Convert absolute value functions into piecewise functions. • Use piecewise functions in real world applications.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
1.5 Parent of Functions	• Look at symmetry characteristics of different functions (even, odd, or neither). • Recognize 12 basic functions (constant, identity, rational, quadratic, cubic, square root, absolute value, cube root, exponential, logarithmic, greatest integer function, signum function, sine function, cosine function, tangent	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2

	function, cotangent function, cosecant function, secant function). • Analyze properties of functions.		
1.6 Transform at ions of Functions	• Use both rigid (translations, reflections) and non-rigid (stretch, shrink) transformations to sketch graphs given a parent function.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
1.7 Combina tio ns of Functions	• Perform operations on functions. • Find a composition of one function with another function. • Decompose a composite function into the two components of the composition.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2

	• Perform function operations that include function compositions. • Model and solve real-world problems involving function operations and compositions.		
1.8 Functions & Their Inverses	• Determine if a function is one to-one using the horizontal line test. • Show two functions are inverses algebraically or graphically. • Use function notation in real world scenarios. • Read and explain functions related to real-world scenarios. • Put restrictions on a relation to create a function.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2

| 1.9 Modeling with Functions | • Write and solve real-world problems from verbal descriptions.
• Use data to write functions. • Solve problems with and without technology.
• Model and solve real-world scenarios. | • Entry / Exit Tickets
• Think-Pair-Share
• Classwork
• Homework
• Lecture
• Model
• Scavenger Hunt | 2 |

Teacher Notes: This unit will take approximately 22 days, including review (1), quiz (1), and test days (2).

Additional Resources:
  • Khan Academy
  • Delta Math
  • Desmos

Differentiation/Modification Strategies

Students with Disabilities	English Language Learners	Gifted and Talented Students	Students at Risk	504 Students
Consult student IEP	Consult student ELL plan	Enrich assignments	Consult with I & RS	Consult student 504 plan

Unit 2 - Overview

Content Area: Pre-Calculus

Unit Title: Polynomial & Rational Functions

Grade Level: 11 – 12

Core Ideas: Students will use polynomials and rational functions to model real-world scenarios. Students will expand their knowledge of these different types of functions to graph by hand. Students will find and use (where applicable), �-intercepts, the �-intercept, intervals of increase and decrease, intervals of concavity, maxima and minima, and asymptotes to graph functions. Students will solve rational equations and inequalities.

Unit 2 - Standards

Standards (Content and Technology)**:**

CPI#:	Statement:
Performance Expectations (NJSLS)	

N-CN.A.	Perform arithmetic operations with complex numbers.
N-CN.A.1.	Know there is a complex number � such that $�^{\&} = -1$, and every complex number has the form $� + ��$ with � and � real.
N-CN.A.3.	(+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
N-CN.C.	Use complex numbers in polynomial identities and equations.
N-CN.C.8.	(+) Extend polynomial identities to the complex numbers. *For example, rewrite $�^{\&} + 4$ as $(� + 2�)(� - 2�)$.*
N-CN.C.9.	(+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
A-SSE.A.	Interpret the structure of expressions.
A-SSE.A.1.	Interpret expressions that represent a quantity in terms of its context.
A-SSE.A.1.a.	Interpret parts of an expression, such as terms, factors, and coefficients.
A-SSE.B.	Write expressions in equivalent forms to solve problems.
A-SSE.B.3.	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
A-SSE.B.3.a.	Factor a quadratic expression to reveal the zeros of the function it defines.
A-SSE.B.3.b.	Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
A-APR.B.	Understand the relationship between zeros and factors of polynomials.
A-APR.B.2.	Know and apply the Remainder Theorem: For a polynomial $�(�)$ and a number �, the remainder on division $� - �$ is $�(�)$, so $�(�) = 0$ if and only if $(� - �)$ is a factor of $�(�)$.
A-APR.B.3.	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
A-APR.D.	Rewrite rational expressions.
A-APR.D.6.	Rewrite simple rational expressions in different forms; write $\frac{'(")}{*(")}$ in the form $�(�) + \frac{+(")}{*(")}$, where $�(�), �(�), �(�),$ and $�(�)$ are polynomials with the degree of $�(�)$ less than the degree of $�(�)$, using inspection, long division, or, for more complicated examples, a computer algebra system.
A-REI.A.	Understand solving equations as a process of reasoning and explain the reasoning.
A-REI.A.2.	Solve simple rational and radical equations in one variable, and give examples

	showing how extraneous solutions may arise.
A-REI.B.	Solve equations and inequalities in one variable.
A-REI.B.4.	Solve quadratic equations in one variable.

A-REI.B.4.a.	Use the method of completing the square to transform any quadratic equation in $\diamond$ into an equation of the form $(\diamond - \diamond)^{\&} = \diamond$ that has the same solutions. Derive the quadratic formula from this form.
A-REI.B.4.b.	Solve quadratic equations by inspection (e.g., for $\diamond^{\&} = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $\diamond \pm \diamond\diamond$ for real numbers $\diamond$ and $\diamond$.
F-LE.A.	Construct and compare linear and exponential models and solve problems.
F-LE.A.1.	Distinguish between situations that can be modeled with linear functions and with exponential functions.
F-LE.A.1.a.	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
F-LE.A.1.b.	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
F-LE.A.2.	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
F-LE.A.3.	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
Career Readiness, Life Literacies, and Key Skills	
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  • Recognizing strengths in others
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  • Identifying and using stress-management strategies
  • Exhibiting self-discipline and self-motivation
  • Setting personal and collective goals
  • Using planning and organizational skills
Relationship Skills: The abilities to establish and maintain healthy and supportive relationships and to effectively navigate settings with diverse individuals and groups.
  • Practicing teamwork and collaborative problem-solving
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  • Seeking or offering support and help when needed

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RST.11-12.3.	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.

Unit Essential Question(s):	Unit Enduring Understandings:
• What is a polynomial function? • What are the characteristics of a quadratic function?	• Linear equations can be written in several different ways, each highlighting different important characteristics.

- How can quadratic functions be used to model real-world scenarios?
- How can we compare the relative rates of growth of different functions?
- Why do asymptotes exist and how are they related to the end behavior of a function?
- Why are rational functions useful in the real world?
- Are a rational expression and its simplified form equivalent?
- How do we graph polynomial and rational functions?
- Why do extraneous solutions sometimes arise when solving rational equations?

- Quadratic functions can be used to model free-fall motion.
- Power functions can be graphed using the sign of the leading coefficient and the degree of the polynomial (among other characteristics).
- The Intermediate Value Theorem can sometimes be used to find specific �- values of a function.
- Polynomials and rational functions can be graphed by using (where applicable), �- intercepts, the �-intercept, intervals of increase and decrease, intervals of concavity, maxima and minima, and asymptotes to graph functions.
- Polynomials can be divided using long division or synthetic division, when applicable.

Evidence of Learning

Formative Assessments:
- Quizzes
- Homework
- On-the-spot check for understanding activities
- Entry / Exit ticket

Summative/Benchmark Assessment(s):
- Quizzes
- Tests
- Projects

Alternative Assessments:
- Notebook & note checks

Resources/Materials:
- All Things Algebra – Precalculus Curriculum • Jean Adams – Precalculus Curriculum • *Larson Precalculus with Limits* textbook

Key Vocabulary:
- Polynomial
- Average rate of change
- Linear equation
  - General form
  - Slope-intercept form
  - Point-slope form
- Quadratic function
- Parabola
- Perfect square trinomial
- Completing the square
- Velocity
- Power function
- Constant of proportion
- Constant of variation
- Monomial function
- End behavior
- Relative rate of growth
- Coefficient

- Leading term
- Constant
- Even and odd multiplicity
- Increasing and decreasing functions
- Maxima and minima
- Intermediate Value Theorem
- Concavity
- Point of inflection
- Remainder Theorem
- Factor Theorem
- Rational Zeros (Roots) Theorem
- Descartes' Rule of Signs
- Fundamental Theorem of Algebra
- Linear Factorization Theorem
- Conjugate Root Theorem
- Complex number
- Real number
- Rational function
- Asymptote
- Rational equation
- Extraneous solution
- Rational inequality
- Polynomial inequality

Suggested Pacing Guide

Lesson Name/Topic	Student Learning Objective(s)	Suggested Tasks/Activities:	Day(s) to Complete
2.1 Quadratic Functions & Graphs Review	• Review the definition of a polynomial. • Recognize that average rate of change is the same as slope. • Compare and convert between different forms of linear equations (general form, slope intercept form, point-slope form). • Identify a polynomial function. • Identify characteristics of a quadratic. • Complete the square to find the vertex of a quadratic. • Model free-fall motion using quadratics. • Use quadratic functions in real world scenarios. • Analyze position vs. time graphs.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
2.2 Power Functions	• Write, graph, and analyze power functions. • Identify end behavior of power functions. • Look at relative rates of growth by comparing different functions.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2

2.3 Polynomial Functions of Higher Degree	• Review terms, coefficients, leading terms, and constants of polynomial functions. • Determine end behavior of polynomial functions by looking at the degree of the function and the sign of the leading coefficient. • Look at number of turning points in a polynomial function in relation to the number of real zeros. • Look at even and odd multiplicity of functions to determine if a function crosses or bounces off the x-axis. • Review increasing vs. decreasing functions, maxima and minima, concavity, and points of inflection. • Introduce the Intermediate Value Theorem.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
2.4 Real Zeros of Polynomials	• Use the Intermediate Value Theorem to determine if zeros (or other specific x-values) exist on an interval. • Use synthetic division and long division to divide polynomials. • Use the Remainder Theorem to evaluate functions. • Use the Factor Theorem to determine if $(x - c)$ is a factor of $f(x)$. • Use the Rational Roots Theorem to find possible zeros of a rational function. • Use Descartes' Rule of Signs to determine the possible number of positive and negative real roots of a polynomial.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
2.5 Complex Zeros	• Use the Fundamental Theorem of Algebra to determine the	• Entry / Exit Tickets • Think-Pair-Share	2

Additional Resources:
- Khan Academy
- Delta Math
- Desmos

Differentiation/Modification Strategies				
Students with Disabilities	**English Language Learners**	**Gifted and Talented Students**	**Students at Risk**	**504 Students**
Consult student IEP	Consult student ELL plan	Enrich assignments	Consult with I & RS	Consult student 504 plan

Unit 3 - Overview
Content Area: Pre-Calculus
Unit Title: Exponential & Logarithmic Functions
Grade Level: 11 – 12
Core Ideas: Students will use exponential and logarithmic equations in real-world scenarios. Students will use properties of logarithmic functions to simplify expressions and solve equations. Students will analyze exponential graphs by looking at key features.

Unit 3 - Standards	
Standards (Content and Technology):	
CPI#:	**Statement:**
Performance Expectations (NJSLS)	
A-SSE.A.	Interpret the structure of expressions.
A-SSE.A.1.	Interpret expressions that represent a quantity in terms of its context.
A-SSE.A.1.b.	Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $\diamond(1 + \diamond)^{,}$ *as the product of* $\diamond$ *and a factor not depending on* $\diamond$.
A-SSE.B.	Write expressions in equivalent forms to solve problems.
A-SSE.B.3.	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A-SSE.B.3.c.	Use the properties of exponents to transform expressions for exponential functions. *For example, the expression* 1.15^{t} *can be rewritten as* $F1.15^{!"}G^{\$\&-} \approx 1.012^{\$\&-}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is* 15%.
F-IF.C.	Analyze functions using different representations.
F-IF.C.7.	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
F-IF.C.7.e.	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
F-IF.C.8.	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
F-IF.C.8.b.	Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $\lozenge = (1.02)^{t}$, $\lozenge = (0.97)^{t}$, $\lozenge = (1.01)^{\$\&-}$, $\lozenge = (1.2)^{\#!\$}$, *and classify them as representing exponential growth or decay.*
F-BF.B.	Build new functions from existing functions.
F-BF.B.3.	Identify the effect on the graph of replacing $\lozenge(\lozenge)$ by $\lozenge(\lozenge) + \lozenge$, $\lozenge \cdot \lozenge(\lozenge)$, $\lozenge(\lozenge\lozenge)$, and $\lozenge(\lozenge + \lozenge)$ for specific values of $\lozenge$ (both positive and negative); find the value of $\lozenge$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
F-BF.B.5.	(+) Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.
F-LE.A.	Construct and compare linear and exponential models and solve problems.
F-LE.A.1.	Distinguish between situations that can be modeled with linear functions and with exponential functions.
F-LE.A.1.a.	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
F-LE.A.1.c.	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F-LE.A.2.	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
F-LE.A.4.	Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $\lozenge\lozenge^{t} = \lozenge$, where $\lozenge$, $\lozenge$, and $\lozenge$ are numbers and the base $\lozenge$ is 2, 10, or $\lozenge$; evaluate the logarithm using technology.
F-LE.B.	Interpret expressions for functions in terms of the situation they model.

F-LE.B.5.	Interpret the parameters in a linear or exponential function in terms of a context.

Career Readiness, Life Literacies, and Key Skills

9.4.12.CI.1.	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CI.3.	Investigate new challenges and opportunities for personal growth, advancement, and transition.
9.4.12.CT.2.	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.TL.4.	Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem.

Computer Science and Design Thinking

8.1.12.DA.5.	Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
8.1.12.DA.6.	Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.1.12.AP.1.	Design algorithms to solve computational problems using a combination of original and existing algorithms.
8.2.12.ED.6.	Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor).

Intercultural Statements (Amistad, Holocaust, LGBT, etc...)

LGBTQ and Disabilities NJSA 18A:35-4.35	Explore mathematicians in the LGBTQ community, including but not limited to Juliette Bruce, NSF Postdoctoral Fellow at University of California, Berkeley, and Stephen Hawking, former Director of Research at the University of Cambridge.
Amistad Law NJSA 18A 52:16A-88	Explore African American mathematicians and scientists, including but not limited to Martha Euphemia Lofton Haynes, the first African American woman to earn a Ph.D mathematics, and Elbert Frank Cox, the first African American man to earn a Ph.D in mathematics in the world.
Amistad Law NJSA 18A 52:16A-88	Discuss and analyze the movie *Hidden Figures,* the story of female African American mathematicians and engineers who worked for NASA.
Holocaust Law NJSA 18A 18A:35- 28	Explore Jewish mathematicians using the article *"Jewish Mathematicians Who Changed the Course of History"* from jewishjournal.com.

AAPI Law NJSA 18A:25-4.44	Explore Asian-American and Pacific Islander mathematicians and scienctists, including but not limited to Dr. Peter Tsai, inventor of the N95 respirator and Diana Ma, data scientist for the Lakers.

Interdisciplinary Connection

6.1.12.EconE M.6.a.	Determine how supply and demand influenced price and output during the Industrial Revolution.
6.1.12.EconN E.6.a.	Analyze the impact of money, investment, credit, savings, debt, and financial institutions on the development of the nation and the lives of individuals.

6.1.12.EconG E.16.a.	Use quantitative data and other sources to assess the impact of international trade, global business organizations, and overseas competition on the United States economy and workforce.

Social Emotional Learning

Self-Awareness: The abilities to understand one's own emotions, thoughts, and values and how they influence behavior across contexts.
• Having a growth mindset
• Developing interests and a sense of purpose
Social Awareness: The abilities to understand the perspectives of and empathize with others, include those from diverse backgrounds, cultures, & contexts.
• Recognizing strengths in others
• Recognizing situational demands and opportunities
Self-Management: The abilities to manage one's emotions, thoughts, and behaviors effectively in different situations and to achieve goals and aspirations.
• Identifying and using stress-management strategies
• Exhibiting self-discipline and self-motivation
• Setting personal and collective goals
• Using planning and organizational skills
Relationship Skills: The abilities to establish and maintain healthy and supportive relationships and to effectively navigate settings with diverse individuals and groups.
• Practicing teamwork and collaborative problem-solving
• Showing leadership in groups
• Seeking or offering support and help when needed

Companion Standards

NJSLSA.R7.	Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
RH-11-12.7.	Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, qualitatively, as well as in words) in order to address a question or solve a problem.
RST.11-12.3.	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.

Unit Essential Question(s):	**Unit Enduring Understandings:**
• How do you model a quantity that changes regularly over time by the same percentage? • How are exponents and logarithms related? • How are exponential functions and logarithmic functions related?	• You can represent repeated multiplication with a function in the form of $\blacklozenge = \blacklozenge\blacklozenge^{"}$, where $\blacklozenge$ is a positive number other than 1. • The function $\blacklozenge = \blacklozenge^{"}$ and $\blacklozenge = \ln \blacklozenge$ are inverse functions. • Logarithms and exponents have corresponding properties. • You can use logarithms to solve exponential equations and exponents to solve logarithmic equations.

Evidence of Learning

Formative Assessments:
- Quizzes
- Homework
- On-the-spot check for understanding activities
- Entry / Exit ticket

Summative/Benchmark Assessment(s):
- Quizzes
- Tests
- Projects

Alternative Assessments:
- Notebook & note checks

Resources/Materials:	**Key Vocabulary:**
• All Things Algebra – Precalculus Curriculum • Jean Adams – Precalculus Curriculum • *Larson Precalculus with Limits* textbook	• Exponential function • Exponential growth • Exponential decay • Natural base $\blacklozenge$ • Logarithmic function • Natural logarithm • Change of base formula

Suggested Pacing Guide

Lesson Name/To pi c	Student Learning Objective(s)	Suggested Tasks/Activities:	Day(s) to Complete

3.1 Exponential & Logistic Functions	• Identify exponential functions. • Find an exponential function from a table of values. • Solve problems involving exponential growth and decay, and the exponential population equation. • Review the natural base �. • Transform exponential functions. • Look at problems involving logistic growth.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
3.2 Logarithmic Functions & Their Graphs	• Recognize that logarithmic functions and exponential functions are inverses of each other. • Use basic properties of logarithms. • Evaluate logarithmic and exponential expressions with and without a calculator. • Solve simple logarithmic equations. • Graph logarithmic functions. • Translate logarithmic functions.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
3.3 Properties of Logarithmic Functions	• Use properties of logarithms to expand and condense expressions. • Evaluate logarithms by using the change of base formula.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model	2

	• Graph logarithmic functions with base *b*. • Use logarithmic models in real world applications.	• Scavenger Hunt	
3.4 Equation Solving	• Solve exponential and logarithmic equations algebraically. • Solve problems involving applications of exponential and logarithmic expressions.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2

| 3.5 Exponential, Logistic, & Logarithmic Modeling | • Use logistic modeling to solve real-world applications.
• Solve problems involving exponential growth and decay. • Model radioactive decay.
• Solve problems using exponential and logarithmic modeling. | • Entry / Exit Tickets
• Think-Pair-Share
• Classwork
• Homework
• Lecture
• Model
• Scavenger Hunt | 2 |
| 3.6 Analyzing Exponential Graphs | • Recognize that growth and decay are not synonymous with increasing and decreasing.
• Analyze exponential functions. | • Entry / Exit Tickets
• Think-Pair-Share
• Classwork
• Homework
• Lecture
• Model
• Scavenger Hunt | 2 |

Teacher Notes: This unit will take approximately 16 days, including review (1), quiz (1), and test days (2).

Additional Resources:
  • Khan Academy
  • Delta Math
  • Desmos

Differentiation/Modification Strategies

Students with Disabilities	English Language Learners	Gifted and Talented Students	Students at Risk	504 Students
Consult student IEP	Consult student ELL plan	Enrich assignments	Consult with I & RS	Consult student 504 plan

Unit 4 - Overview

Content Area: Pre-Calculus

Unit Title: Trigonometric Functions

Grade Level: 11 – 12

Core Ideas: Students will be introduced to radians and expand their knowledge of trigonometric functions. Students will draw angles in standard position and use right triangle trigonometry to evaluate expressions. Students will learn the unit circle and graphing trigonometric functions. They will model real-world applications using trigonometry.

Unit 4 - Standards

Standards (Content and Technology)**:**

CPI#:	Statement:
Performance Expectations (NJSLS)	
F-IF.C.	Analyze functions using different representations.
F-IF.C.7.	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
F-IF.C.7.e.	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline and amplitude.
F-TF.A.	Extend the domain of trigonometric functions using the unit circle.
F-TF.A.1.	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
F-TF.A.2.	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
F-TF.A.3.	(+) Use special right triangles to determine geometrically the values of sine, cosine, tangent for $/_!, /_0$, and $\bullet/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\bullet\bullet, \bullet + \bullet, 2\bullet - \bullet$ in terms of their values for $\bullet$, where $\bullet$ is any real number.
F-TF.A.4.	(+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
F-TF.B.	Model periodic phenomena with trigonometric functions.
F-TF.B.5.	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
F-TF.B.6.	(+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
F-TF.B.7.	(+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
F-TF.C.	Prove and apply trigonometric identities.
F-TF.C.8.	Prove the Pythagorean identity $\sin^{\&}\bullet + \cos^{\&}\bullet = 1$ and use it to find $\sin\bullet$, $\cos\bullet$, or $\tan\bullet$ given $\sin\bullet$, $\cos\bullet$, or $\tan\bullet$ and the quadrant of the angle.
Career Readiness, Life Literacies, and Key Skills	
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9.4.12.CT.2.	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
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Computer Science and Design Thinking

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  • Developing interests and a sense of purpose
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  • Recognizing strengths in others
  • Recognizing situational demands and opportunities
Self-Management: The abilities to manage one's emotions, thoughts, and behaviors effectively in different situations and to achieve goals and aspirations.
  • Identifying and using stress-management strategies
  • Exhibiting self-discipline and self-motivation
  • Setting personal and collective goals

  • Using planning and organizational skills
Relationship Skills: The abilities to establish and maintain healthy and supportive relationships and to effectively navigate settings with diverse individuals and groups.
  • Practicing teamwork and collaborative problem-solving
  • Showing leadership in groups
  • Seeking or offering support and help when needed

Companion Standards

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RST.11-12.3.	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.

Unit Essential Question(s):
- What is the relationship between degrees and angles?
- How are angles and trigonometric functions used to model and solve real-world problems?
- How does the unit circle relate to trigonometric functions?
- How do you evaluate trigonometric functions of acute angles?
- How do you use trigonometric functions to model real-life data?
- How do you evaluate inverse trigonometric functions?

Unit Enduring Understandings:
- An angle with a full circle rotation measures 2� radians. An angle with a semicircle rotation measures � radians. • The cosine function corresponds with the �-coordinate of the point where the terminal side of the angle intersects the unit circle. The sine function corresponds with the �-coordinate of the point where the terminal side of the angle intersects the unit circle.
- Trigonometric identities can be used to simplify and evaluate expressions.
- Trigonometric identities are true in both radian and degree modes.
- The graphs of trigonometric functions relate to the trigonometric values on the unit circle.
- The inverse sine (for example) function � = arcsin � or � = sin$^{\%\$}$ �, can be stated as the phrase, "the angle whose sine is �".
- The range of each inverse trigonometric function is limited to allow it to be a function.
- Trigonometry can be used to model and solve real-world applications.

Evidence of Learning

Formative Assessments:
- Quizzes
- Homework
- On-the-spot check for understanding activities
- Entry / Exit ticket

Summative/Benchmark Assessment(s):
- Quizzes
- Tests
- Projects

Alternative Assessments:
- Notebook & note checks

Resources/Materials:	Key Vocabulary:
• All Things Algebra – Precalculus Curriculum • Jean Adams – Precalculus Curriculum • *Larson Precalculus with Limits* textbook	• Vertex • Initial side of an angle • Terminal side of an angle • Standard position of an angle • Coterminal angle • Reference angle • Degrees-minutes-seconds • Radian • Central angle • Unit circle • Sine • Cosine • Tangent • Cotangent • Secant • Cosecant • Special right triangles • Arc length • Linear velocity (speed) • Angular velocity (speed) • Sector • Unit circle • Quadrantal angle • Odd and even trigonometric functions • Domain • Range • Periodic function • Trigonometric identity • Reciprocal identity • Quotient identity • Fundamental identity • Pythagorean identity • Amplitude • Period • Phase shift • Vertical shift • Midline • Inverse • One-to-one • Exponent notation • Arc notation • Composite function • Angle of elevation • Angle of depression • Indirect measurement

Suggested Pacing Guide			
Lesson Name/Topi pi c	**Student Learning Objective(s)**	**Suggested Tasks/Activities:**	**Day(s) to Complete**
4.1 Angles & Their Measures	• Look at angle characteristics (vertex, initial side, terminal side). • Graph angles in standard position. • Find coterminal angles. • State reference angles. • Convert between degrees minutes-seconds to decimal degrees. • Understand that radians are a unitless measure used to measure angles. • Identify central angles on the unit circle. • Convert between degrees and radians.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
4.2 Right Triangle Trigono met ry	• Review the six basic trigonometric functions and the two special right triangles. • Use one trigonometric ratio to find the others. • Find trigonometric values with and without a calculator. • Use trigonometric functions in real-world applications.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
4.3 Arc Length & Circular Motion	• Find the arc length of an intercepted arc. • Find linear and angular velocity. • Find linear and angular speed. • Find area of a sector. • Find distance between two angle measures (e.g., the distance between two cities on a map).	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2

4.4 The Unit Circle	• Identify relationships that exist on the unit circle. • Use special right triangles to find trigonometric values. • Find trigonometric functions using the unit circle.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2

	• Determine if trigonometric functions are even or odd.		
4.5 Properties of Trigono met ric Functions	• Find ratios of trigonometric functions on the unit circle. • Find the domain and range of trigonometric functions. • Recognize the periods of the six trigonometric functions. • Find exact values of trigonometric functions. • Use reciprocal and quotient identities to simplify expressions and find trigonometric values. • Use the Pythagorean Identities to simplify expressions and find trigonometric values.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
4.6 Graphs of Sine & Cosine Functions	• Graph the sine and cosine functions and identify the characteristics of each function, including the domain, range, �- and �-intercepts, maximum and minimum, symmetry, and key points. • Identify and use the phase shift and vertical shift to graph sine and cosine functions.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2

4.7 Graphs of Other Trigono met ric Functions	• Graph the tangent, cotangent, secant, and cosecant functions and identify the characteristics of each function, including the domain, range, �- and �- intercepts, maximum and minimum, symmetry, and key points. • Identify and use the phase shift and vertical shift to graph tangent, cotangent, secant, and cosecant functions.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
4.8 Inverse Trigono met ric Functions	• Find the domain of inverse trigonometric functions using the range of trigonometric functions. • Find inverse trigonometric values, both with and without a calculator. • Solve real-world applications using inverse trigonometric functions.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2

4.9 More Inverse Trigono met ric Functions	• Use both exponent notation and arc notation when writing and evaluating inverse functions. • Use the properties of inverses to identify domains and ranges of functions. • Graph inverse trigonometric functions. • Find inverse trigonometric values, both with and without a calculator. • Find exact values of composite functions. • Approximate the value of inverse trigonometric functions.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
4.10 Sinusoidal Curve Fitting and Applicatio ns	• Graph trigonometric functions with transformations. • Find sinusoidal functions from data.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2

| 4.11 Modeling with Trigono met ry | • Model real-world applications using trigonometry, including angles of elevation and depression, indirect measurements, distance between two points, navigation, and simple harmonic motion. | • Entry / Exit Tickets
• Think-Pair-Share
• Classwork
• Homework
• Lecture
• Model
• Scavenger Hunt | 2 |

Teacher Notes: This unit will take approximately 26 days, including review (1), quiz (1), and test days (2).

Additional Resources:
  • Khan Academy
  • Delta Math
  • Desmos

Differentiation/Modification Strategies

Students with Disabilities	English Language Learners	Gifted and Talented Students	Students at Risk	504 Students
Consult student IEP	Consult student ELL plan	Enrich assignments	Consult with I & RS	Consult student 504 plan

Unit 5 - Overview

Content Area: Pre-Calculus

Unit Title: Analytic Trigonometry

Grade Level: 11 – 12

Core Ideas: Students will prove trigonometric identities. Students will use the Laws of Sine and Cosine to solve oblique triangles. They will solve trigonometric equations.

Unit 5 - Standards

Standards (Content and Technology)**:**

CPI#:	Statement:
Performance Expectations (NJSLS)	
F-TF.C.	Prove and apply trigonometric identities.
F-TF.C.8.	Prove the Pythagorean identity $\sin^2 \theta + \cos^2 \theta = 1$ and use it to find sin �, cos �, or tan � given sin �, cos �, or tan � and the quadrant of the angle.

F-TF.C.9.	(+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Career Readiness, Life Literacies, and Key Skills

9.4.12.CI.1.	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CI.3.	Investigate new challenges and opportunities for personal growth, advancement, and transition.
9.4.12.CT.2.	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.TL.4.	Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem.

Computer Science and Design Thinking

8.1.12.DA.5.	Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
8.1.12.DA.6.	Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.1.12.AP.1.	Design algorithms to solve computational problems using a combination of original and existing algorithms.
8.2.12.ED.6.	Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor).

Intercultural Statements (Amistad, Holocaust, LGBT, etc...)

LGBTQ and Disabilities NJSA 18A:35-4.35	Explore mathematicians in the LGBTQ community, including but not limited to Juliette Bruce, NSF Postdoctoral Fellow at University of California, Berkeley, and Stephen Hawking, former Director of Research at the University of Cambridge.
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Amistad Law NJSA 18A 52:16A-88	Discuss and analyze the movie *Hidden Figures,* the story of female African American mathematicians and engineers who worked for NASA.
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Interdisciplinary Connection (include SEL)

6.1.12.EconE M.6.a.	Determine how supply and demand influenced price and output during the Industrial Revolution.
6.1.12.EconN E.6.a.	Analyze the impact of money, investment, credit, savings, debt, and financial institutions on the development of the nation and the lives of individuals.
6.1.12.EconG E.16.a.	Use quantitative data and other sources to assess the impact of international trade, global business organizations, and overseas competition on the United States economy and workforce.

Social Emotional Learning

Self-Awareness: The abilities to understand one's own emotions, thoughts, and values and how they influence behavior across contexts.
　• Having a growth mindset
　• Developing interests and a sense of purpose
Social Awareness: The abilities to understand the perspectives of and empathize with others, include those from diverse backgrounds, cultures, & contexts.
　• Recognizing strengths in others
　• Recognizing situational demands and opportunities
Self-Management: The abilities to manage one's emotions, thoughts, and behaviors effectively in different situations and to achieve goals and aspirations.
　• Identifying and using stress-management strategies
　• Exhibiting self-discipline and self-motivation
　• Setting personal and collective goals
　• Using planning and organizational skills
Relationship Skills: The abilities to establish and maintain healthy and supportive relationships and to effectively navigate settings with diverse individuals and groups.
　• Practicing teamwork and collaborative problem-solving
　• Showing leadership in groups
　• Seeking or offering support and help when needed

Companion Standards

NJSLSA.R7.	Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
RH-11-12.7.	Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, qualitatively, as well as in words) in order to address a question or solve a problem.
RST.11-12.3.	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.

Unit Essential Question(s):	Unit Enduring Understandings:
• What is an identity? • When do you use trigonometric identities to evaluate, simplify, verify / prove, and solve trigonometric functions, expressions, identities, and equations, respectively? • How do you use trigonometric identities to evaluate, simplify, verify / prove, and solve	• Identities are two equivalent expressions. When verifying identities, either side must be worked on separately as an expression. • Trigonometric identities must be used to solve certain types of equations. • Trigonometric identities can be used to evaluate, simplify, verify / prove, and solve

trigonometric functions, expressions, identities, and equations, respectively? • When do the Laws of Sine and Cosine apply in real-world situations?	trigonometric functions, expressions, identities, and equations, respectively. • There are many real-world applications associated with the various types of trigonometric identities.

Evidence of Learning

Formative Assessments:
- Quizzes
- Homework
- On-the-spot check for understanding activities
- Entry / Exit ticket

Summative/Benchmark Assessment(s):
- Quizzes
- Tests
- Projects

Alternative Assessments:
- Notebook & note checks

Resources/Materials:	Key Vocabulary:
• All Things Algebra – Precalculus Curriculum • Jean Adams – Precalculus Curriculum • *Larson Precalculus with Limits* textbook	• Trigonometric identity • Reciprocal identity • Quotient identity • Pythagorean identity • Unit circle • Even-odd identity • Co-function identity • Sum and difference identity • Reduction formula • Double-angle identity • Half-angle identity • Power-reducing formula • Law of sines • Law of cosines • Ambiguous case • Heron's formula

Suggested Pacing Guide

Lesson Name/To pi c	Student Learning Objective(s)	Suggested Tasks/Activities:	Day(s) to Complete
5.1 Fundame nta l Identities	• Use the reciprocal identities, quotient identities, and Pythagorean identities to simplify and factor expressions. • Determine which functions are even and odd. • Use the co-function identities to simplify expressions.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
5.2 Proving Trigono metric Identities	• Prove trigonometric identities.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
5.3 Sum & Difference Identities	• Simplify expressions and prove identities using sum and difference formulas. • Prove reduction formulas.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2

5.4 Multiple Angles Identities	• Use double-angle identities, half-angle identities, and power-reducing identities to prove trigonometric identities. • Rewrite trigonometric expressions by reducing the power of 4. • Solve trigonometric equations algebraically that have double angles and half angles.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
5.5 Law of Sines	• Solve triangles using the Law of Sines. • Find area of oblique triangles. • Solve problems involving real world applications of the Law of Sines.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
5.6 Law of Cosines	• Solve triangles using the Law of Cosines. • Find area of triangles using Heron's Formula. • Solve problems involving real world applications of the Law of Cosines.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
5.7 Solving Trigono met ric Equations	• Solve trigonometric equations graphically. • Solve trigonometric equations by collecting like terms, square roots, factoring, and by using identities to rewrite the equation first.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2

Teacher Notes: This unit will take approximately 18 days, including review (1), quiz (1), and test days (2).

Additional Resources:
  • Khan Academy
  • Delta Math

  • Desmos

Differentiation/Modification Strategies

Students with Disabilities	English Language Learners	Gifted and Talented Students	Students at Risk	504 Students
Consult student IEP	Consult student ELL plan	Enrich assignments	Consult with I & RS	Consult student 504 plan

Unit 6 - Overview

Content Area: Pre-Calculus

Unit Title: Polar Coordinates & Vectors

Grade Level: 11 – 12

Core Ideas: Students will be introduced to the polar coordinate system. Students will use vectors, parametric equations, and polar coordinates to model real-world scenarios.

Unit 6 - Standards

Standards (Content and Technology):

CPI#:	Statement:
Performance Expectations (NJSLS)	
N-CN.B.	Represent complex numbers and their operations on the complex plane.
N-CN.B.4.	(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
N-CN.B.5.	(+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $,-1 + \sqrt{3}\lozenge{-}^{!}= 8$ *because* $,-1 + \sqrt{3}\lozenge{-}$ *has modulus* 2 *and argument* 120°.
N-CN.B.6.	(+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
N-VM.A.	Represent and model with vector quantities.
N-VM.A.1	(+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\lozenge$, $\lvert\lozenge\rvert$, $\lVert\lozenge\rVert$).
N-VM.A.2.	(+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

N-VM.A.3.	(+) Solve problems involving velocity and other quantities that can be represented by vectors.
N-VM.B.	Perform operations on vectors.
N-VM.B.4.	(+) Add and subtract vectors.
N-VM.B.4.a.	Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
N-VM.B.4.b.	Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
N-VM.B.4.c.	Understand vector subtraction � − � as � + (−�), where −� is the additive inverse of �, with the same magnitude as � and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
N-VM.B.5.	(+) Multiply a vector by a scalar.
N-VM.B.5.a.	Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $�,�_"$, $�_1 - = ,��_"$, $��_1 -$.
N-VM.B.5.b.	Compute the magnitude of a scalar multiple �� using $\|��\| = |�|�$. Compute the direction of �� knowing that when $|�|� \neq 0$, the direction of �� is either along � (for $� > 0$) or against � (for $� < 0$).
Career Readiness, Life Literacies, and Key Skills	
9.4.12.CI.1.	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CI.3.	Investigate new challenges and opportunities for personal growth, advancement, and transition.

9.4.12.CT.2.	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.TL.4.	Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem.
Computer Science and Design Thinking	
8.1.12.DA.5.	Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
8.1.12.DA.6.	Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.

8.1.12.AP.1.	Design algorithms to solve computational problems using a combination of original and existing algorithms.
8.2.12.ED.6.	Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor).

Intercultural Statements (Amistad, Holocaust, LGBT, etc...)

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Interdisciplinary Connection

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Companion Standards

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RST.11-12.3.	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.

Unit Essential Question(s):	Unit Enduring Understandings:
• Why are functions and relations represented by vectors? • Why are functions and relations represented by parametric equations? • Why are functions and relations represented by polar equations?	• Functions and relations can be represented using vectors, parametric equations, and polar coordinates. • Vectors, parametric equations, and polar coordinates are useful in solving real world applications.

Evidence of Learning

Formative Assessments:
- Quizzes
- Homework
- On-the-spot check for understanding activities
- Entry / Exit ticket

Summative/Benchmark Assessment(s):
- Quizzes
- Tests
- Projects

Alternative Assessments:
- Notebook & note checks

Resources/Materials:	Key Vocabulary:
• All Things Algebra – Precalculus Curriculum • Jean Adams – Precalculus Curriculum • *Larson Precalculus with Limits* textbook	• Polar coordinate • Pole • Polar axis • Directed distance • Directed angle • Polar system • Rectangular coordinate • Cardioid • Limacon

	• Rose curve • Logarithmic spiral • Complex numbers • Complex plane • Real axis • Imaginary axis • DeMoivre's Theorem • Vector • Magnitude • Unit vector • Vector component • Dot product • Orthogonal vector • Projection • Work

Suggested Pacing Guide

Lesson Name/Topi pi c	Student Learning Objective(s)	Suggested Tasks/Activities:	Day(s) to Complete

6.1 Polar Coordinates	• Use polar language such as the polar axis, polar coordinates, directed distance, and directed angle. • Graph using the polar coordinate system. • Find all polar coordinates of a single point. • Convert between polar and rectangular coordinates.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
6.2 Polar Graphs & Equations	• Graph polar graphs (cardioids, limacons, and rose curves). • Find symmetry of different polar graphs. • Find zeros and maximum θ values of polar graphs. • Graph a logarithmic spiral.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
6.3 More About Polar Graphs	• Determine the domain restrictions for sketching the polar curve through one cycle. • Find the intersections of two polar graphs.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
6.4 The Complex Plane; De Moivre's Theorem	• Graph complex numbers. • Find the absolute value of complex numbers. • Find the polar form of a complex number.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture	2

	• Convert between polar and trigonometric forms. • Find the product and quotient of complex numbers. • Use DeMoivre's Theorem to simplify complex numbers. • Find θ-th roots of complex numbers.	• Model • Scavenger Hunt	

6.5 Vectors in the Plane	• Represent vectors geometrically. • Translate vectors. • Write vectors in component form. • Write vectors as a linear combination. • Find the magnitude of a vector. • Use the properties of vectors to perform operations on vectors. • Find unit vectors. • Determine if two vectors are equal. • Solve real-world applications involving vectors.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
6.6 The Dot Product	• Find the dot product of vectors. • Use properties of dot products. • Find the angle between two vectors. • Show that vectors are orthogonal. • Find the projection of vectors. • Find the work done.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2

Teacher Notes: This unit will take approximately 16 days, including review (1), quiz (1), and test days (2).

Additional Resources:
- Khan Academy
- Delta Math
- Desmos

Differentiation/Modification Strategies

Students with Disabilities	Students with Disabilities	Students with Disabilities	Students with Disabilities	Students with Disabilities
Consult student IEP	Consult student IEP	Consult student IEP	Consult student IEP	Consult student IEP

Unit 7 - Overview

Content Area: Pre-Calculus
Unit Title: Systems of Equations & Matrices
Grade Level: 11 – 12

Core Ideas: Students will solve both linear and nonlinear systems of equation. Students will perform matrix operations and find the inverse of a matrix. Students will find the solution to systems of inequalities.	

<h2 style="text-align:center">Unit 7 - Standards</h2>

Standards (Content and Technology):

CPI#:	Statement:
Performance Expectations (NJSLS)	
N-VM.C.	Perform operations on matrices and use matrices in applications.
N-VM.C.6.	(+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
N-VM.C.7.	(+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
N-VM.C.8.	(+) Add, subtract, and multiply matrices of appropriate dimensions.
N-VM.C.9.	(+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
N-VM.C.10.	(+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
N-VM.C.11.	(+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
N-VM.C.12.	(+) Work with 2 � 2 matrices as a transformation of the plane, and interpret the absolute value of the determinant in terms of area.
A-CED.A.	Create equations that describe numbers or relationships.
A-CED.A.3.	Represent constraints by equations or inequalities, and by systems of equations and / or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
A-REI.C.	Solve systems of equations.
A-REI.5.	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
A-REI.6.	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A-REI.7.	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*
A-REI.8.	(+) Represent a system of linear equations as a single matrix equation in a vector variable.
A-REI.9.	(+) Find the inverse of a matric if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).
A-REI.D.	Represent and solve equations and inequalities graphically.
A-REI.D.10.	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A-REI.D.11.	Explain why the x-coordinate of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions

	approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and / or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
A-REI.D.12.	Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
Career Readiness, Life Literacies, and Key Skills	
9.4.12.CI.1.	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CI.3.	Investigate new challenges and opportunities for personal growth, advancement, and transition.
9.4.12.CT.2.	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.TL.4.	Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem.
Computer Science and Design Thinking	
8.1.12.DA.5.	Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
8.1.12.DA.6.	Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.1.12.AP.1.	Design algorithms to solve computational problems using a combination of original and existing algorithms.

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Interdisciplinary Connection (include SEL)

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RST.11-12.3.	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.

Unit Essential Question(s):
- What methods do you have to solve systems of equations and what are the advantages of each method?
- How do we find the inverse of a matrix and when does a matrix not have an inverse?
- How can we use systems of inequalities to model and solve real-world applications?

Unit Enduring Understandings:
- A system of linear equations can be expressed as a matrix and that matrix can sometimes be used to solve that system.
- An inverse matrix can be used to solve a system of linear equations.
- Matrix multiplication is not commutative.

Evidence of Learning

Formative Assessments:
- Quizzes
- Homework
- On-the-spot check for understanding activities
- Entry / Exit ticket

Summative/Benchmark Assessment(s):
- Quizzes
- Tests

- Projects

Alternative Assessments:
- Notebook & note checks

Resources/Materials:	Key Vocabulary:
• All Things Algebra – Precalculus Curriculum • Jean Adams – Precalculus Curriculum • *Larson Precalculus with Limits* textbook	• Linear system • System of equations • Solution • Consistent • Independent • Inconsistent • Dependent • Elementary row operation • Gaussian elimination • Matrix • Order of a matrix • Augmented matrix • Reduced row-echelon form • Inverse • Determinant • Minor matrix • Cofactor • Partial fraction • Decomposition • Feasible region • Linear programming

Suggested Pacing Guide			
Lesson Name/Topic	Student Learning Objective(s)	Suggested Tasks/Activities:	Day(s) to Complete

7.1 Solving Systems of Equations	• Categorize systems of equations by name (consistent and independent, inconsistent, and consistent and dependent). • Solve non-linear systems. • Solve systems with three variables. • Use systems for curve fitting applications. • Use systems for real-world applications.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
7.2 Multivari abl e Linear Systems	• Solve systems of linear equations using elementary row operations. • Look at the graphical interpretation of three-variable systems. • Solve dependent systems of equations.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2

	• Use systems of equations for real-world applications.		
7.3 Matrices & Systems of Equations	• Determine the order of a matrix. • Write an augmented matrix. • Perform row operations. • Solve systems of equations using Gauss-Jordan elimination. • Solve systems of equations with infinite solutions and no solution.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
7.4 The Algebra of Matrices	• Find the sum and difference of two matrices. • Perform operations on matrices. • Use matrices in real-world applications.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2

7.5 Inverses of Matrices & Matrix Equations	• Find the inverse of a matrix. • Solve systems of equations with inverse matrices. • Use real-world applications and inverse matrices to solve problems.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
7.6 The Determinant of a Square Matrix	• Find the determinant of a square matrix. • Find the inverse of a matrix, if it exists.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
7.7 Partial Fractions	• Decompose rational expressions into partial fractions. • Write partial decompositions of rational expressions.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
7.8 Systems of Nonlinear Equations	• Solve nonlinear systems of equations using substitution, elimination, graphically, and with technology.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
7.9 Systems of	• Graph quadratic inequalities.	• Entry / Exit Tickets • Think-Pair-Share	2

Inequalities & Linear Programming	• Find minimum and maximum values of functions using linear programming. • Solve real world applications, such as maximizing profits or finding the optimal cost.	• Classwork • Homework • Lecture • Model • Scavenger Hunt	

Teacher Notes: This unit will take approximately 22 days, including review (1), quiz (1), and test days (2).

Additional Resources:
  • Khan Academy
  • Delta Math

• Desmos

Differentiation/Modification Strategies				
Students with Disabilities	**English Language Learners**	**Gifted and Talented Students**	**Students at Risk**	**504 Students**
Consult student IEP	Consult student ELL plan	Enrich assignments	Consult with I & RS	Consult student 504 plan

Unit 8 - Overview	
Content Area: Pre-Calculus	
Unit Title: Conic Sections	
Grade Level: 11 – 12	
Core Ideas: Students will graph equations of circles, parabolas, ellipses, and hyperbolas. Students will write equations of each given characteristics. Students will use the properties of each conic to model and solve problems. Students will use polar coordinates to represent and solve problems involving conic sections.	
Unit 8 - Standards	
Standards (Content and Technology):	
CPI#:	**Statement:**
Performance Expectations (NJSLS)	
G-GPE.A.	Translate between the geometric description and the equation for a conic section.
G-GPE.A.1.	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
G-GPE.A.2.	Derive the equation of a parabola given a focus and directrix.
G-GPE.A.3.	(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
Career Readiness, Life Literacies, and Key Skills	
9.4.12.CI.1.	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CI.3.	Investigate new challenges and opportunities for personal growth, advancement, and transition.
9.4.12.CT.2.	Explain the potential benefits of collaborating to enhance critical thinking and

	problem solving.
9.4.12.TL.4.	Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem.

Computer Science and Design Thinking

8.1.12.DA.5.	Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
8.1.12.DA.6.	Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.1.12.AP.1.	Design algorithms to solve computational problems using a combination of original and existing algorithms.
8.2.12.ED.6.	Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor).

Intercultural Statements (Amistad, Holocaust, LGBT, etc...)

LGBTQ and Disabilities NJSA 18A:35-4.35	Explore mathematicians in the LGBTQ community, including but not limited to Juliette Bruce, NSF Postdoctoral Fellow at University of California, Berkeley, and Stephen Hawking, former Director of Research at the University of Cambridge.
Amistad Law NJSA 18A 52:16A-88	Explore African American mathematicians and scientists, including but not limited to Martha Euphemia Lofton Haynes, the first African American woman to earn a Ph.D mathematics, and Elbert Frank Cox, the first African American man to earn a Ph.D in mathematics in the world.
Amistad Law NJSA 18A 52:16A-88	Discuss and analyze the movie *Hidden Figures,* the story of female African American mathematicians and engineers who worked for NASA.
Holocaust Law NJSA	Explore Jewish mathematicians using the article *"Jewish Mathematicians Who Changed the Course of History"* from jewishjournal.com.

18A 18A:35-28	
AAPI Law NJSA 18A:25-4.44	Explore Asian-American and Pacific Islander mathematicians and scienctists, including but not limited to Dr. Peter Tsai, inventor of the N95 respirator and Diana Ma, data scientist for the Lakers.

Interdisciplinary Connection

6.1.12.Eco	Determine how supply and demand influenced price and output during the

nE M.6.a.	Industrial Revolution.
6.1.12.Eco nN E.6.a.	Analyze the impact of money, investment, credit, savings, debt, and financial institutions on the development of the nation and the lives of individuals.
6.1.12.Eco nG E.16.a.	Use quantitative data and other sources to assess the impact of international trade, global business organizations, and overseas competition on the United States economy and workforce.

Social Emotional Learning

Self-Awareness: The abilities to understand one's own emotions, thoughts, and values and how they influence behavior across contexts.
  • Having a growth mindset
  • Developing interests and a sense of purpose
Social Awareness: The abilities to understand the perspectives of and empathize with others, include those from diverse backgrounds, cultures, & contexts.
  • Recognizing strengths in others
  • Recognizing situational demands and opportunities
Self-Management: The abilities to manage one's emotions, thoughts, and behaviors effectively in different situations and to achieve goals and aspirations.
  • Identifying and using stress-management strategies
  • Exhibiting self-discipline and self-motivation
  • Setting personal and collective goals
  • Using planning and organizational skills
Relationship Skills: The abilities to establish and maintain healthy and supportive relationships and to effectively navigate settings with diverse individuals and groups.
  • Practicing teamwork and collaborative problem-solving
  • Showing leadership in groups
      Seeking or offering support and help when needed

Companion Standards

NJSLSA.R7.	Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
RH-11-12.7.	Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, qualitatively, as well as in words) in order to address a question or solve a problem.
RST.11-12.3.	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.

Unit Essential Question(s):	Unit Enduring Understandings:
• How do you recognize each conic section and solve problems involving different conic sections? • How do you classify a conic from its general equation?	• Circles, parabolas, ellipses, and hyperbolas are all closely related.

Evidence of Learning

Formative Assessments:
- Quizzes
- Homework
- On-the-spot check for understanding activities
- Entry / Exit ticket

Summative/Benchmark Assessment(s):
- Quizzes
- Tests
- Projects

Alternative Assessments:
- Notebook & note checks

Resources/Materials:	Key Vocabulary:
• All Things Algebra – Precalculus Curriculum • Jean Adams – Precalculus Curriculum • *Larson Precalculus with Limits* textbook	• Circle • Parabola • Ellipse • Hyperbola • Point • Single line • Intersecting lines • Directrix • Axis of symmetry • Vertex • Focus • Focal length • Focal width • Latus rectum • Locus of points • Center • Chord • Circumference • Diameter • Arc • Radius • Sector • Tangent of a circle • Area • Minor axis • Semi-minor axis • Major axis • Semi-major axis • Eccentricity • Transverse axis • Semi-transverse axis • Conjugate axis • Semi-conjugate axis • Parametric equation

Suggested Pacing Guide

Lesson Name/Topic	Student Learning Objective(s)	Suggested Tasks/Activities:	Day(s) to Complete

8.1 Parabolas	• Graph parabolas. • Find the vertex, focus, and directrix of a parabola. • Write equations of parabolas with given characteristics.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
8.2 Circles	• Graph circles. • Find the center and radius of a circle. • Write equations of circles with given characteristics.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
8.3 Ellipses	• Graph ellipses. • Find the vertices, co-vertices, and foci of an ellipse. • Identify the major and minor aces of an ellipse. • Write equations of ellipses with given characteristics. • Find the eccentricity of ellipses.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
8.4 Hyperbolas	• Graph hyperbolas. • Find the center, vertices, and asymptotes of a hyperbola. • Identify the transverse, conjugate, semi-transverse, and semi-conjugate axes of hyperbolas. • Write equations of hyperbolas with given characteristics.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
8.5 Parametric Equations	• Graph parametric curves. • Find the rectangular equation of a parametric curve. • Find parametric equations from given conditions.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2

8.6 Parametric Applications	• Graph parametric curves. • Use parametric equations in real-world applications.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2

Teacher Notes: This unit will take approximately 16 days, including review (1), quiz (1), and test days (2).

Additional Resources:
- Khan Academy
- Delta Math
- Desmos

Differentiation/Modification Strategies

Students with Disabilities	English Language Learners	Gifted and Talented Students	Students at Risk	504 Students
Consult student IEP	Consult student ELL plan	Enrich assignments	Consult with I & RS	Consult student 504 plan

Unit 9 – Overview

Content Area: Pre-Calculus

Unit Title: Sequences & Series

Grade Level: 11 – 12

Core Ideas: Students will study and analyze the patterns of sequences and series. They will represent sequences and series in different ways, find the �-th term of different sequences, and will use summation notation.

Unit 9 - Standards

Standards (Content and Technology):

CPI#:	Statement:
Performance Expectations (NJSLS)	
A-SSE.B.	Write expressions in equivalent forms to solve problems.
A-SSE.B.4.	Derive and / or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*

F-IF.A.	Understand the concept of a function and use function notation.
F-IF.A.3.	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1, f(n + 1) = f(n) + f(n - 1)$ *for* $n \geq 1$.
F-BF.A.	Build a function that models a relationship between two quantities.
F-BF.A.2.	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
F-LE.A.	Construct and compare linear and exponential models and solve problems.
F-LE.A.2.	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
Career Readiness, Life Literacies, and Key Skills	
9.4.12.CI.1.	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CI.3.	Investigate new challenges and opportunities for personal growth, advancement, and transition.
9.4.12.CT.2.	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.TL.4.	Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem.
Computer Science and Design Thinking	
8.1.12.DA.5.	Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
8.1.12.DA.6.	Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.1.12.AP.1.	Design algorithms to solve computational problems using a combination of original and existing algorithms.
8.2.12.ED.6.	Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor).
Intercultural Statements (Amistad, Holocaust, LGBT, etc...)	
LGBTQ and Disabilities NJSA 18A:35-4.35	Explore mathematicians in the LGBTQ community, including but not limited to Juliette Bruce, NSF Postdoctoral Fellow at University of California, Berkeley, and Stephen Hawking, former Director of Research at the University of Cambridge.

Amistad Law NJSA 18A 52:16A-88	Explore African American mathematicians and scientists, including but not limited to Martha Euphemia Lofton Haynes, the first African American woman to earn a Ph.D mathematics, and Elbert Frank Cox, the first African American man to earn a Ph.D in mathematics in the world.
Amistad Law NJSA 18A 52:16A-88	Discuss and analyze the movie *Hidden Figures,* the story of female African American mathematicians and engineers who worked for NASA.
Holocaust Law NJSA 18A 18A:35-28	Explore Jewish mathematicians using the article *"Jewish Mathematicians Who Changed the Course of History"* from jewishjournal.com.
AAPI Law NJSA 18A:25-4.44	Explore Asian-American and Pacific Islander mathematicians and scientists, including but not limited to Dr. Peter Tsai, inventor of the N95 respirator and Diana Ma, data scientist for the Lakers.
Interdisciplinary Connection	
6.1.12.EconE M.6.a.	Determine how supply and demand influenced price and output during the Industrial Revolution.
6.1.12.EconN E.6.a.	Analyze the impact of money, investment, credit, savings, debt, and financial institutions on the development of the nation and the lives of individuals.
6.1.12.EconG E.16.a.	Use quantitative data and other sources to assess the impact of international trade, global business organizations, and overseas competition on the United States economy and workforce.
Social Emotional Learning	

Self-Awareness: The abilities to understand one's own emotions, thoughts, and values and how they influence behavior across contexts.
- Having a growth mindset
- Developing interests and a sense of purpose

Social Awareness: The abilities to understand the perspectives of and empathize with others, include those from diverse backgrounds, cultures, & contexts.
- Recognizing strengths in others
- Recognizing situational demands and opportunities

Self-Management: The abilities to manage one's emotions, thoughts, and behaviors effectively in different situations and to achieve goals and aspirations.
- Identifying and using stress-management strategies
- Exhibiting self-discipline and self-motivation
- Setting personal and collective goals
- Using planning and organizational skills

Relationship Skills: The abilities to establish and maintain healthy and supportive relationships and to effectively navigate settings with diverse individuals and groups.
- Practicing teamwork and collaborative problem-solving
- Showing leadership in groups
- Seeking or offering support and help when needed

Companion Standards

NJSLSA.R7.	Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
RH-11-12.7.	Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, qualitatively, as well as in words) in order to address a question or solve a problem.

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RST.11-12.3.	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.

Unit Essential Question(s):	**Unit Enduring Understandings:**
• How do you represent a sequence of numbers or the sum of a sequence? • How do you find the �-th term or partial sum of an arithmetic sequence? • How do you find terms and sums of geometric sequences?	• Represent sequences and series, including with summation notation. • Model and find sums of arithmetic and geometric sequences.

Evidence of Learning

Formative Assessments:
- Quizzes
- Homework
- On-the-spot check for understanding activities
- Entry / Exit ticket

Summative/Benchmark Assessment(s):
- Quizzes
- Tests
- Projects

Alternative Assessments:
- Notebook & note checks

Resources/Materials:
- All Things Algebra – Precalculus Curriculum
- Jean Adams – Precalculus Curriculum •
- *Larson Precalculus with Limits* textbook

Key Vocabulary:
- Sequence
- Infinite sequence
- Term
- Finite sequence
- Recursive formula
- Explicit rule
- Factorial notation
- Summation notation
- Index
- Upper limit
- Lower limit
- Arithmetic sequence
- Arithmetic mean
- Convergence
- Divergence
- Geometric sequence
- Common ratio
- Mathematical induction

Suggested Pacing Guide

Lesson Name/ Topic	Student Learning Objective(s)	Suggested Tasks/Activities:	Day(s) to Complete
9.1 Sequences & Summation Notation	• Write the terms of a sequence given an explicit rule.	• Entry / Exit Tickets • Think-Pair-Share • Classwork	2

	• Use technology to graph a sequence. • Write a recursive formula. • Find the first few terms from a recursive formula. • Use summation notation to evaluate the sum. • Use sequences and summation notation in real world contexts.	• Homework • Lecture • Model • Scavenger Hunt	
9.2 Arithmetic Sequences & Partial Sums	• Find the general term of a sequence. • Find the sum of the first ◆ terms of an arithmetic sequence. • Find the sum of a series. • Determine whether a sequence converges or diverges. • Find the limit of a sequence. • Use explicit formulas to find a specified term of a sequence.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
9.3 Geometric Sequences & Series	• Determine whether a given sequence is geometric. • Find specified term for geometric series. • Find the sum of a geometric sequence. • Find and use the geometric mean in a sequence.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
9.4 Mathematical Induction	• Use the principle of mathematical induction to prove statements.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2

Teacher Notes: This unit will take approximately 12 days, including review (1), quiz (1), and test days (2).

Additional Resources:
  • Khan Academy
  • Delta Math
  • Desmos

Differentiation/Modification Strategies

Students with Disabilities	English Language Learners	Gifted and Talented Students	Students at Risk	504 Students
Consult student IEP	Consult student ELL plan	Enrich assignments	Consult with I & RS	Consult student 504 plan

Unit 10 - Overview

Content Area: Pre-Calculus

Unit Title: Introduction to Calculus

Grade Level: 11 – 12

Core Ideas: Students will be introduced to limits. They will find limits graphically, numerically, and algebraically. Students will find limits at infinity and infinite limits. Students will find simple derivatives.

Unit 10 - Standards

Standards (Content and Technology):

CPI#:	Statement:
Performance Expectations (NJSLS)	
*Please refer to College Board's Unit 1 of the AP Calculus Course Exam Description (CED).	
Career Readiness, Life Literacies, and Key Skills	
9.4.12.CI.1.	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CI.3.	Investigate new challenges and opportunities for personal growth, advancement, and transition.
9.4.12.CT.2.	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.TL.4.	Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem.
Computer Science and Design Thinking	
8.1.12.DA.5.	Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
8.1.12.DA.6.	Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.1.12.AP.1.	Design algorithms to solve computational problems using a combination of original

	and existing algorithms.
8.2.12.ED.6.	Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor).

Intercultural Statements (Amistad, Holocaust, LGBT, etc...)

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Amistad Law NJSA 18A 52:16A-88	Discuss and analyze the movie *Hidden Figures,* the story of female African American mathematicians and engineers who worked for NASA.
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AAPI Law NJSA 18A:25-4.44	Explore Asian-American and Pacific Islander mathematicians and scientists, including but not limited to Dr. Peter Tsai, inventor of the N95 respirator and Diana Ma, data scientist for the Lakers.

Interdisciplinary Connection

6.1.12.EconE M.6.a.	Determine how supply and demand influenced price and output during the Industrial Revolution.
6.1.12.EconN E.6.a.	Analyze the impact of money, investment, credit, savings, debt, and financial institutions on the development of the nation and the lives of individuals.
6.1.12.EconG E.16.a.	Use quantitative data and other sources to assess the impact of international trade, global business organizations, and overseas competition on the United States economy and workforce.

Social Emotional Learning

Self-Awareness: The abilities to understand one's own emotions, thoughts, and values and how they influence behavior across contexts.
- Having a growth mindset
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- Exhibiting self-discipline and self-motivation
- Setting personal and collective goals
- Using planning and organizational skills

Relationship Skills: The abilities to establish and maintain healthy and supportive relationships and to effectively navigate settings with diverse individuals and groups.
- Practicing teamwork and collaborative problem-solving
- Showing leadership in groups
- Seeking or offering support and help when needed

Companion Standards

NJSLSA.R7.	Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
RH-11-12.7.	Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, qualitatively, as well as in words) in order to address a question or solve a problem.
RST.11-12.3.	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.

Unit Essential Question(s):
- How do you find and interpret the limit of a function?
- How do you evaluate limits that cannot be solved through the use of direct substitution?

Unit Enduring Understandings:
- Use techniques for calculating the limit of a graph at a given value of �.

Evidence of Learning

Formative Assessments:
- Quizzes
- Homework
- On-the-spot check for understanding activities
- Entry / Exit ticket

Summative/Benchmark Assessment(s):
- Quizzes

- Tests
- Projects

Alternative Assessments:
- Notebook & note checks

Resources/Materials:	**Key Vocabulary:**
• *Larson Precalculus with Limits* textbook • All Things Algebra – Precalculus Curriculum • Jean Adams – Precalculus Curriculum	• One-sided limit • Left-hand limit • Right-hand limit • General limit • Limit at a point • Continuity • Limit notation • Limit Existence Theorem

Suggested Pacing Guide

Lesson Name/Topic	Student Learning Objective(s)	Suggested Tasks/Activities:	Day(s) to Complete
10.1 Introduction to Limits	• Find one-sided and general limits. • Use limit notation. • Find a limit from a table (numerically). • Recognize limits that do not exist. • Find limits graphically.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
10.2 Finding Limits Analytically	• Use properties of limits to evaluate limits. • Use direct substitution, factor and cancel, rationalization technique, and special trigonometric limits to find limits.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
10.3 Limits at Infinity & Continuity	• Find limits at infinity and infinite limits. See how these relate to horizontal and vertical asymptotes. • Find limits of rational functions. • Determine on what intervals a function is continuous.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2

10.4 Tangent Lines & the Derivative	• Find and interpret average rate of change, instantaneous rate of change, average velocity, and instantaneous velocity. • Find the slope of a tangent line using the limit process. • Use the difference quotient. • Find the derivative at a point.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2
	• Solve problems involving particle motion and velocity.		
10.5 Derivatives & The Power Rule	• Find the derivative of functions using the power rule. • Find a derivative at a point.	• Entry / Exit Tickets • Think-Pair-Share • Classwork • Homework • Lecture • Model • Scavenger Hunt	2

Teacher Notes: This unit will take approximately 14 days, including review (1), quiz (1), and test days (2).

Additional Resources:
• Khan Academy
• Delta Math
• Desmos

Differentiation/Modification Strategies

Students with Disabilities	English Language Learners	Gifted and Talented Students	Students at Risk	504 Students
Consult student IEP	Consult student ELL plan	Enrich assignments	Consult with I & RS	Consult student 504 plan